3.5 Solving Proportions

Essential Question How can you use ratio tables and cross products to solve proportions in science?

1 ACTIVITY: Solving a Proportion in Science

SCIENCE Scientists use *ratio tables* to determine the amount of a compound (like salt) that is dissolved in a solution. Work with a partner to show how scientists use cross products to determine the unknown quantity in a ratio.

a. Sample: Salt Water

| Salt Water | 1 L | 3 L |
|------------|-------|-----|
| Salt | 250 g | хg |

$$\frac{3\cancel{L}}{1\cancel{L}} = \frac{xg}{250g}$$

Write proportion.





$$3 \cdot 250 = 1 \cdot x$$

Set cross products equal.

$$750 = x$$

Simplify.

So, there are 750 grams of salt in the 3-liter solution.

b. White Glue Solution

| Water | 1/ _{2 cup} | 1 cup |
|------------|---------------------|--------|
| White Glue | 1/ _{2 cup} | x cups |

c. Borax Solution

| Borax | 1 tsp | 2 tsp |
|-------|-------|--------|
| Water | 1 cup | x cups |

d. Slime (see recipe)

| Borax Solution | 1/2 cup | 1 cup |
|---------------------|---------|--------|
| White Glue Solution | y cups | x cups |

Recipe for SLIME

- Add ¹/₂ cup of water and ¹/₂ cup white glue. Mix thoroughly. This is your white glue solution.
- 2. Add a couple drops of food coloring to the glue solution. Mix thoroughly.
- 3. Add 1 teaspoon of borax to 1 cup of water. Mix thoroughly. This is your borax solution (about 1 cup).
- 4. Pour the borax solution and the glue solution into a separate bowl.
- Place the slime that forms in a plastic bag and squeeze the mixture repeatedly to mix it up.

ACTIVITY: The Game of Criss Cross

Preparation:

- Cut index cards to make 48 playing cards.
- Write each number on a card.

1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 10, 10, 10, 12, 12, 12, 13, 13, 13, 14, 14, 14, 15, 15, 15, 16, 16, 16, 18, 20, 25

• Make a copy of the game board.

To Play:

- Play with a partner.
- Deal 8 cards to each player.
- Begin by drawing a card from the remaining cards. Use four of your cards to try to form a proportion.
- Lay the four cards on the game board. If you form a proportion, say "Criss Cross" and you earn 4 points. Place the four cards in a discard pile. Now it is your partner's turn.
- If you cannot form a proportion, then it is your partner's turn.
- When the original pile of cards is empty, shuffle the cards in the discard pile and start again.
- The first player to reach 20 points wins.

What Is Your Answer?

- **3. IN YOUR OWN WORDS** How can you use ratio tables and cross products to solve proportions in science? Give an example.
- **4. PUZZLE** Use each number once to form three proportions.

1 2 10 4 12 20

15 5 16 6 8 3

Practice

Use what you discovered about solving proportions to complete Exercises 10–13 on page 126.

CRISS CROSS





Solving Proportions

Method 1 Use mental math. (Section 3.4)

Method 2 Use the Multiplication Property of Equality. (Section 3.5)

Method 3 Use the Cross Products Property. (Section 3.5)

Solving Proportions Using Multiplication **EXAMPLE**

Solve
$$\frac{5}{7} = \frac{x}{21}$$
.

$$\frac{5}{7} = \frac{x}{21}$$

Write the proportion.

$$21 \cdot \frac{5}{7} = 21 \cdot \frac{x}{21}$$

Multiply each side by 21.

$$15 = x$$

Simplify.

The solution is 15.

On Your Own



Solve the proportion using multiplication.

1.
$$\frac{w}{6} = \frac{6}{9}$$

2.
$$\frac{12}{10} = \frac{a}{15}$$
 3. $\frac{y}{6} = \frac{2}{4}$

3.
$$\frac{y}{6} = \frac{2}{4}$$

Solving Proportions Using the Cross Products Property EXAMPLE

Solve each proportion.

a.
$$\frac{x}{8} = \frac{7}{10}$$

b.
$$\frac{9}{y} = \frac{3}{17}$$

$$x \cdot 10 = 8 \cdot 7$$

Use the Cross Products Property.

$$9 \cdot 17 = y \cdot 3$$

$$10x = 56$$

Multiply.

$$153 = 3y$$

$$x = 5.6$$

Divide.

$$51 = y$$

The solution is 5.6.

The solution is 51.





Solve the proportion using the Cross Products Property.

4.
$$\frac{2}{7} = \frac{x}{28}$$

5.
$$\frac{12}{5} = \frac{6}{y}$$

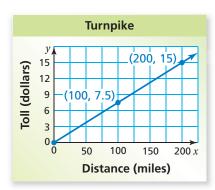
6.
$$\frac{40}{z+1} = \frac{15}{6}$$

EXAMPLE

Real-Life Application

The toll due on a turnpike is proportional to the number of miles driven. How much does it cost to drive 150 miles?

TOLL PLAZA
1/2 MILE
REDUCE SPEED



Method 1: Interpret the slope as a unit rate.

slope =
$$\frac{\text{change in } y}{\text{change in } x}$$

= $\frac{7.5}{100}$ Substitute.
= 0.075 Divide.

The unit rate is \$0.075 per mile. Multiply to find the total cost.

$$150 \, \text{mir} \cdot \frac{\$0.075}{1 \, \text{mir}} = \$11.25$$

:• It costs \$11.25 to drive 150 miles on the turnpike.

Method 2: Write and solve a proportion.

$$\frac{7.5}{100} = \frac{x}{150}$$
 dollars
$$\frac{7.5}{100} = \frac{x}{150}$$
 Use (100, 7.5) to write a proportion.
$$\frac{7.5}{100} = \frac{x}{150}$$
 Multiply each side by 150.
$$11.25 = x$$
 Simplify.

: It costs \$11.25 to drive 150 miles on the turnpike.

On Your Own

7. WHAT IF? In Example 3, how much does it cost to drive 75 miles on the turnpike?

Exercises





Vocabulary and Concept Check

- 1. WRITING What are three ways you can solve a proportion?
- **2. OPEN-ENDED** Which way would you choose to solve $\frac{3}{r} = \frac{6}{14}$? Explain your reasoning.
- **3. NUMBER SENSE** Does $\frac{x}{4} = \frac{15}{3}$ have the same solution as $\frac{x}{15} = \frac{4}{3}$? Use the Cross Products Property to explain your answer



Practice and Problem Solving

Solve the proportion using multiplication.

1 4.
$$\frac{9}{5} = \frac{z}{20}$$

5.
$$\frac{h}{15} = \frac{16}{3}$$

6.
$$\frac{w}{4} = \frac{42}{24}$$

7.
$$\frac{35}{28} = \frac{n}{12}$$

8.
$$\frac{7}{16} = \frac{x}{4}$$

9.
$$\frac{y}{9} = \frac{44}{54}$$

Solve the proportion using the Cross Products Property.

2 **10.**
$$\frac{a}{6} = \frac{15}{2}$$

11.
$$\frac{10}{7} = \frac{8}{k}$$

11.
$$\frac{10}{7} = \frac{8}{k}$$
 12. $\frac{3}{4} = \frac{\nu}{14}$

13.
$$\frac{5}{n} = \frac{16}{32}$$

14.
$$\frac{36}{42} = \frac{24}{r}$$

14.
$$\frac{36}{42} = \frac{24}{r}$$
 15. $\frac{9}{10} = \frac{d}{6.4}$ **16.** $\frac{x}{8} = \frac{3}{12}$ **17.** $\frac{8}{m} = \frac{6}{15}$

16.
$$\frac{x}{8} = \frac{3}{12}$$

17.
$$\frac{8}{m} = \frac{6}{15}$$

18.
$$\frac{4}{24} = \frac{c}{36}$$

19.
$$\frac{20}{16} = \frac{d}{12}$$

20.
$$\frac{30}{20} = \frac{w}{14}$$

18.
$$\frac{4}{24} = \frac{c}{36}$$
 19. $\frac{20}{16} = \frac{d}{12}$ **20.** $\frac{30}{20} = \frac{w}{14}$ **21.** $\frac{2.4}{1.8} = \frac{7.2}{k}$

22. ERROR ANALYSIS Describe and correct the error in solving the proportion $\frac{m}{8} = \frac{15}{24}$.



$$\frac{m}{8} = \frac{15}{24}$$

$$8 \cdot m = 24 \cdot 1$$

 $m = 45$

- **23. PENS** Forty-eight pens are packaged in four boxes. How many pens are packaged in nine boxes?
- **24. PIZZA PARTY** How much does it cost to buy 10 medium pizzas?

3 Medium Pizzas for \$10.50

Solve the proportion.

25.
$$\frac{2x}{5} = \frac{9}{15}$$

26.
$$\frac{5}{2} = \frac{d-2}{4}$$

27.
$$\frac{4}{k+3} = \frac{8}{14}$$

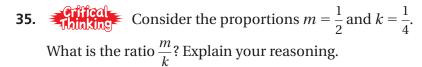
28. TRUE OR FALSE? Tell whether the statement is *true* or *false*. Explain.

If
$$\frac{a}{b} = \frac{2}{3}$$
, then $\frac{3}{2} = \frac{b}{a}$.

- **29. CLASS TRIP** It costs \$95 for 20 students to visit an aquarium. How much does it cost for 162 students?
- **30. GRAVITY** A person who weighs 120 pounds on Earth weighs 20 pounds on the moon. How much does a 93-pound person weigh on the moon?



- **31. HAIR** The length of human hair is proportional to the number of months it has grown.
 - **a.** How long does it take hair to grow 8 inches?
 - **b.** Use a different method than the one in part (a) to find how long it takes hair to grow 20 inches.
- **32. CHEETAH** Cheetahs are the fastest mammals in the world. They can reach speeds of 70 miles per hour.
 - **a.** At this speed, how long would it take a cheetah to run 17 miles?
 - **b. RESEARCH** Use the Internet or library to find how long a cheetah can maintain a speed of 70 miles per hour.
- **33. AUDIENCE** There are 144 people in an audience. The ratio of adults to children is 5 to 3. How many are adults?
- **34. LAWN SEED** Three pounds of lawn seed covers 1800 square feet. How many bags are needed to cover 8400 square feet?





A

Fair Game Review What you learned in previous grades & lessons

Copy and complete. (Skills Review Handbook)

38.
$$56 \text{ oz} = 16$$

39.
$$1\frac{1}{2}$$
 mi = ft

- **40. MULTIPLE CHOICE** How many cups of milk are shown? *(Skills Review Handbook)*
 - $\bigcirc A \quad \frac{7}{10}$
- \bigcirc $\frac{7}{8}$
- © $1\frac{3}{4}$ c
- (**D**) 14 (

